Exercises for Introduction to Quantum Computing (physics7514) Wise 2025

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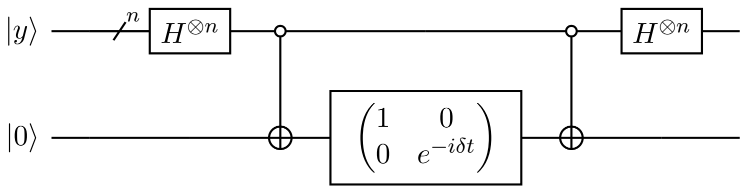
1 Grover's algorithm

a) Show that the following circuit



implements the operation $e^{-i|\psi\rangle\langle\psi|\delta t}$ with

$$|\psi\rangle \;=\; \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle \,, \quad N = 2^n,$$

as introduced in the lecture. Notice that the action of the controlled operation is

$$|0\rangle^{\otimes n} |0\rangle \to |0\rangle^{\otimes n} |1\rangle \;, \qquad |0\rangle^{\otimes n} |1\rangle \to |0\rangle^{\otimes n} |0\rangle \,,$$

and it is the identity for every other ket *(4 points)*.

b) Consider Grover's algorithm as introduced in the lecture. $|x_s\rangle$ is the solution state and

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle = \alpha|x_s\rangle + \beta|x_s^{\perp}\rangle \,.$$

What is the largest value of $N = 2^n$ for which one knows the solution with certainty after only one application of the oracle? Explain why! *(3 points)*

2 Error mitigation

Consider the exercise 1 of exercise sheet 11, where a two-qubit Hamiltonian,

$$H = X_0 \otimes Y_1, \tag{1}$$

is used to evolve the state $|00\rangle$ for $\delta t = 1$.

a) Compute $\langle\psi| Z_0 \otimes Z_1 |\psi\rangle$ with $|\psi\rangle = e^{-iH} |00\rangle$ *(2 points)*.

b) In Qiskit, measure $\langle\psi| Z_0 \otimes Z_1 |\psi\rangle$ using a noisy simulator and

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options.resilience_level = i
```

with `i=0` (no error mitigation), `i=1` (readout error mitigation), `i=2` (zero-noise extrapolation), `i=3` (probabilistic error cancellation).
To use this option, you need to use the primitives of Qiskit. Please see the attached notebook, where the measurement of $\langle\psi| Z_0 \otimes Z_1 |\psi\rangle$ is implemented. Which value of `options.resilience_level` produces the most accurate result *(1 point)*?