Exercises for Introduction to Quantum Computing (physics7514) Wise 2025

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1 Quantum Simulation on Quantum Hardware

Consider the following two-qubit Hamiltonian:

$$H = X_0 \otimes Y_1. \tag{1}$$

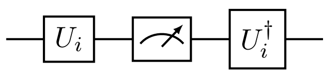
a) Construct a quantum circuit to simulate the Hamiltonian, by performing the unitary transform $e^{-i\delta t H}$ for any δt. The circuit can contain only one- and two-qubit gates *(3 points)*.

b) Implement the quantum circuit using the statevector approach discussed in Sheet 10.1 c) in Qiskit and evolve the initial state $|\psi(0)\rangle = |00\rangle$ for $\delta t = 1$. Obtain the outcome probabilities for each basis state of the combined system of qubit 0 and 1.*(3 points)*.

c) In part b), you classically simulated the quantum computation using an ideal, noise-free simulator and directly accessed the wave function. Now, repeat this computation on a simulated noisy quantum computer using FakeAlmadenV2 as the `backend`, with the remaining simulation setup the same as introduced in Sheet 1.1 *(2 points)*.

2 Pauli Measurements

The task of measuring the eigenvalues of the three Pauli matrices $\sigma_i = X, Y, Z$ is quite common. Pauli matrices are both hermitian and unitary. In quantum computing, they can be interpreted both as gates or as observables. Consider the following single-qubit circuit:



where U_i is a unitary transformation. Note that after the measurement operation, the qubit is either in the state $|0\rangle$ or in the state $|1\rangle$.

a) For each Pauli matrix $\sigma_i = X, Y, Z$, find out how to choose the unitary transformation U_i such that the final state after the circuit is an eigenstate of $\sigma_i = X, Y, Z$, i.e., the measurement outcome indicates one of the two eigenvalues of the corresponding σ_i matrix *(1 point)*.

b) Next, we aim to generalise the single-qubit case from a) to two-qubit measurements, i.e., we aim to measure expectation values of tensor products of $\sigma_i \otimes \sigma_j$ with $\sigma_i \in \{1, X, Y, Z\}$ and $i \in \{1, 2\}$ labelling the qubit. How do you need to choose the two-qubit unitary transformation U_i for measuring the expectation values of $Z \otimes Z$, $1 \otimes Y$, $X \otimes Z$, and $X \otimes Y$ *(1 point)*?