Exercises for Introduction to Quantum Computing (physics7514) Wise 2025

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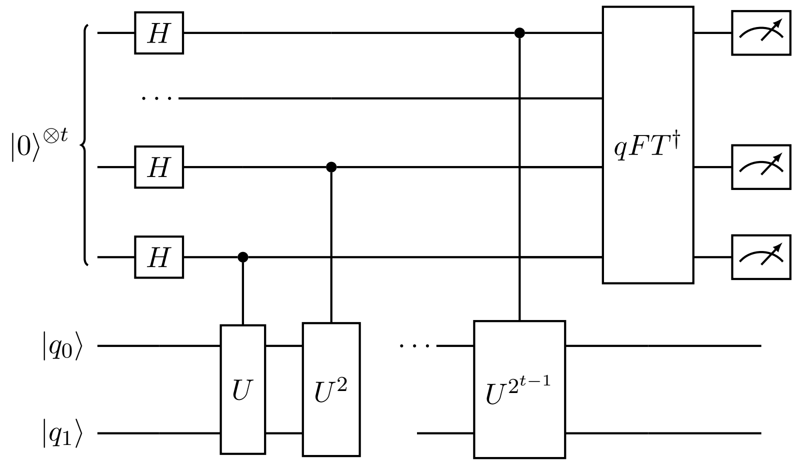
1 Phase estimation algorithm

Consider a unitary operator U acting on a two-qubit state, $U|q_0q_1\rangle = Z|q_0\rangle \otimes S|q_1\rangle$, where $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the Pauli-Z gate and $S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ is the phase gate. We want to measure the eigenvalues of U using the phase estimation algorithm. For this, we need a first register of t qubits and a second register of two qubits. The operator U is applied to the second register. The circuit for the phase estimation algorithm is:



a) What are the eigenvalues and the eigenvectors of the operator U *(1 point)*?

b) What is the minimal number of qubits t in the first register that is required to *exactly* measure the eigenvalues *(2 points)*?

c) Initialize the second register in the state $|q_0q_1\rangle = |01\rangle$ and implement the phase estimation algorithm for the operator U in Qiskit, such that you measure the correct eigenvalue with probability 1 *(3 points)*.

d) Repeat the phase estimation algorithm in Qiskit, but this time with initializing the second register in the state $|q_0q_1\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$. Compute the probability of measuring a phase of $\phi = \frac{1}{4}$ analytically, as well as using your Qiskit implementation*(2 points)*.

e) Finally, consider the phase estimation algorithm with only *one* qubit in the first register, while the second register is initialized in the state $|q_0 q_1\rangle = |01\rangle$. What are the probabilities p_0 and p_1 of measuring the outcome 0 and 1, respectively *(2 points)*?