Exercises for Introduction to Quantum Computing (physics7514) Wise 2025

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1 Quantum Fourier Transform (continued)

As on the previous exercise sheet, consider a 3-qubit system with basis $|j\rangle = |j_0 j_1 j_2\rangle$, where $j = j_0 2^2 + j_1 2^1 + j_2 2^0 = 0, 1, ..., 7$ labels the $N = 2^3 = 8$ basis states. The Quantum Fourier Transform (qFT) acts on the basis elements as

$$\text{qFT}\,|j\rangle = \frac{1}{\sqrt{8}} \sum_{k=0}^{7} e^{2\pi i \frac{jk}{8}} |k\rangle \ .$$

Now, consider the state

$$|\psi(x)\rangle = \frac{1}{\sqrt{8}} \sum_{k=0}^{7} e^{2\pi i \frac{kx}{8}} |k\rangle \ .$$

a) Compute the result of applying the inverse $\text{qFT}^\dagger$ to the state $|\psi(x = 1)\rangle$ *(1 point)*.

b) Compute the probability of measuring 0 after a measurement of the state $\text{qFT}^\dagger\,|\psi(x = \frac{1}{2})\rangle$ *(4 points)*.

c) Compute the probability of measuring either 0 or 1 (in binary 001) after a measurement of the state $\text{qFT}^\dagger\,|\psi(x = \frac{1}{2})\rangle$ *(1 point)*.

d) Using Qiskit:

- prepare the state $|\psi(x = \frac{1}{2})\rangle$ *(2 points)*,
- apply the $\text{qFT}^\dagger$ *(1 point)*,
- measure all the qubits for 10000 times and print the result of the measurements (or plot the histogram) *(1 point)*.

Hints: To prepare the state $|\psi(x = \frac{1}{2})\rangle$, it may be useful to rewrite it as

$$|\psi(x)\rangle = \frac{1}{\sqrt{8}} \left(|0\rangle + e^{2\pi i \frac{1}{2}\frac{1}{2}} |1\rangle\right) \left(|0\rangle + e^{2\pi i \frac{1}{2}\frac{1}{4}} |1\rangle\right) \left(|0\rangle + e^{2\pi i \frac{1}{2}\frac{1}{8}} |1\rangle\right) \ .$$