Exercises for Introduction to Quantum Computing (physics7514) Wise 2025

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1 Quantum Fourier Transform

Consider a 3-qubit system with basis $|j\rangle = |j_0 j_1 j_2\rangle$, where $j = j_0 2^2 + j_1 2^1 + j_2 = 0, 1, ..., 7$ labels the $N = 2^3 = 8$ basis states. The quantum Fourier Transform (qFT) acts on the basis elements as

$$\text{qFT}\,|j\rangle = \frac{1}{\sqrt{8}} \sum_{k=0}^{7} e^{2\pi i \frac{jk}{8}} |k\rangle \ .$$

Note that Qiskit and most QC textbooks use the abbreviation "QFT" instead of "qFT", but we will use "QFT" for quantum field theory later when discussing physics applications.

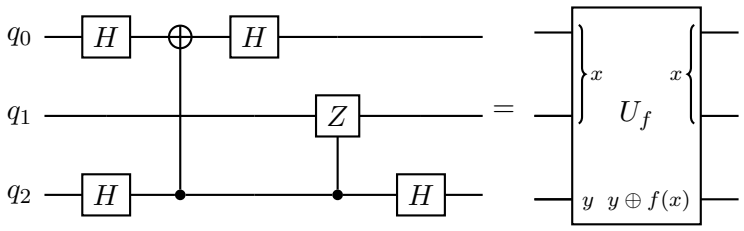
- Find a state $|\phi\rangle$ such that $\text{qFT}^\dagger\,|\phi\rangle = |100\rangle$, with $\text{qFT}^\dagger$ being the inverse qFT *(1 point).*
- Compute the probability of measuring the output 100 from the state $|\phi\rangle$. Without doing further calculations, tell the probability of measuring any output from $|\phi\rangle$. Explain the result using the uncertainty principle. *(1 point).*
- In the lecture, we derived a quantum circuit for the qFT. Find a quantum circuit for the inverse qFT *(2 points).*

2 Deutsch–Jozsa algorithm

In the lecture, we discussed a solution to the Deutsch problem. Consider now the case of x being an n-qubit and y being a 1-qubit register. This is the Deutsch–Jozsa problem, where we aim to find out if the function $f\colon \{0,1\}^n \to \{0,1\}$ is constant (i.e., either 0 on all inputs or 1 on all inputs) or balanced (i.e., 1 for one half of the inputs and 0 for the other half).

- For $n = 2$, show that the following quantum circuit implements an oracle U_f with

$$U_f\,|q_0 q_1 q_2\rangle = |q_0 q_1 (q_2 \oplus f(q_0, q_1))\rangle \quad \textit{(2 points).}$$



- Is the function f represented by this oracle balanced or constant *(2 points)*?
- Implement the Deutsch–Jozsa algorithm for $n = 2$ in Qiskit and use it to find out if the oracle is balanced or constant *(2 points)*.